NAME:

Spring 2020 Math 1201 Exam 1

Instructions: WRITE YOUR NAME CLEARLY. Do as many problems as you can for a maximal score of 100. SHOW YOUR WORK!

1.

a) Suppose $\lim_{h\to 0} \frac{f(2+h)-f(2)}{h} = 10.$

Find $\lim_{h\to 0} \frac{f(2+5h)-f(2)}{h}$ [5 pts]

b) Suppose $\lim_{x\to 3}(f(x)+g(x)) = 6$ and $\lim_{x\to 3}(f(x)-g(x)) = 4.$

Find $\lim_{x\to 3} g(x)$ [5 pts]

2. Use the squeeze theorem to evaluate $\lim_{x\to 0} x^2\, Sin(e^x) Cos\left(\frac{1}{x}\right)$

[10 pts]

3. Evaluate $\lim_{x\to-\infty} \frac{\sqrt{x^2+2}}{3x-6}$ [10 pts]

4. Find an equation of the tangent line to the curve $y = \frac{xCos(x)}{1+x}$ at the point (0, 0). [10 pts]

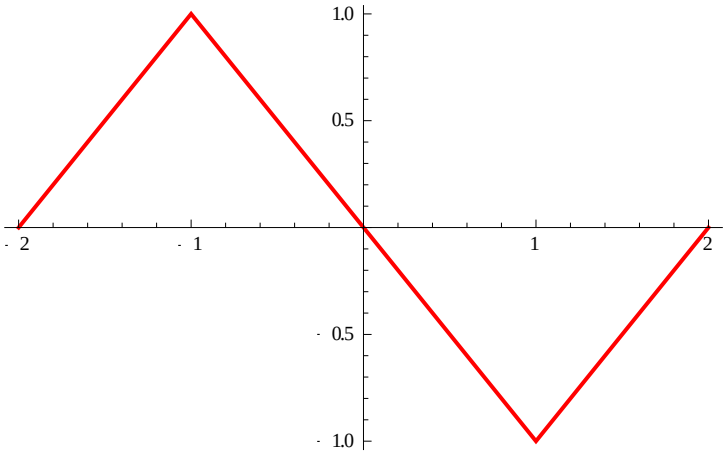
5. Find the derivative of the function $f(x) = \sqrt{1-3x}$ using the definition of the derivative at any given point x. [10 pts]

6. Let $f(x) = \begin{cases} \dfrac{x^2 - x}{x^2 - 1} & \text{if } x \neq 1 \\ 1 & \text{if } x = 1 \end{cases}$. Determine the points where f is discontinuous. Justify your answer. [10 pts]

7. Use the intermediate value theorem to show that the equation $x^3 - 4x + 1 = 0$ has at least one solution in the interval [0, 1]. [10 pts]

8. Find a value k that will make $f(x) = \begin{cases} 7x - 2 & \text{if } x \le 1 \\ kx^2 & \text{if } x > 1 \end{cases}$. [10 pts]

9. The graph of the function $y = f(x)$ is displayed below



Draw the graph of $y = f\,'(x)$. [10 pts]

10. Evaluate $\lim_{\theta \to 0} \frac{sin(5\theta)\tan(3\theta)}{\theta^2}$ [10 pts]

Extra-Credit

11. Prove by means of a delta-epsilon argument that $\lim_{x \to 2}(x^2 - 4x + 5) = 1$ [10 pts]

12. Calculate $\lim_{h\to 0} \frac{Cos(x+h) - Cos(x-h)}{h}$ [10 pts]